3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Physics

Course : PHYS1011 (MAJOR)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. Answer *any five* questions: 2×5=10

(a) Write down the Taylor series expansion of a function $f(x)$ about $x = x_0$. Prove that:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$$

(b) Solve the equation: $\frac{dy}{dx} + lnx^y = 0$

(c) The vector equation: $\vec{A} \times \vec{C} = \vec{B} \times \vec{C}$ does not necessarily imply $\vec{A} = \vec{B}$. Justify your answer.

(d) Prove the identity $\vec{\nabla}.(\vec{A} \times \vec{B}) = \vec{B}.(\vec{\nabla} \times \vec{A}) - \vec{A}.(\vec{\nabla} \times \vec{B})$.

(e) A force $\vec{F}$ is defined as: $\vec{F} = (3x + 5y)\hat{\imath} + (5x + 2z)\hat{\jmath} + (2y + 3x + 4z)\hat{k}$. Is the force conservative?

(f) Write the expression of an elementary volume in terms of scale factors and co-ordinates. Hence calculate the volume of a sphere of radius a.

(g) $\phi(x, y, z)$ = Constant, describes a surface. Prove that $\vec{\nabla}\phi$ is perpendicular to the surface.

(h) The equation of motion of a particle of mass m is: $m\frac{d^2\vec{r}}{dt^2} = \hat{r}f(r)$. Prove that $\vec{r} \times \frac{d\vec{r}}{dt}$ = Constant.

Group-B

Answer *any two* questions: 5×2=10

2. Evaluate the integral: $\int_s \vec{A}.\hat{n}ds$ where $\vec{A} = 2xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and s are the surfaces of the unit cube $0 \leq x, y, z \leq 1$. 5

3. Solve the differential equation: $x\frac{dy}{dx} + y = x^2y^2$. 5

4. If $u = f(x - y, y - z, z - x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 5

5. Find the unit vectors in cylindrical co-ordinate system and prove that they are orthogonal to one another. 5

Group-C

Answer *any two* questions: 10×2=20

6. (a) Find the differential equation whose solution is $xy = Ae^{x} + Be^{-x} + x^{2}$. 5

(b) Solve the differential equation: $(x^{3} - x)\frac{dy}{dx} - (3x^{2} - 1)y = x^{5} - 2x^{3} + x$. 5

7. (a) Evaluate the integral: $\iiint \frac{dxdydz}{x^{2}+y^{2}+z^{2}}$ throughout the volume of the sphere $x^{2} + y^{2} + z^{2} = a^{2}$. 5

(b) Apply Green's theorem in a plane to evaluate the integral: $\oint_{C} (3x^{2} - 8y^{2})dx + (4y - 6xy)dy$, where C is the region bounded by $y = \sqrt{x}$ and $y = x^{2}$. 5

8. (a) Divide 120 into three parts so that the sum of their products taken two at a time shall be the maximum. 5

(b) Define Jacobian, $J\left(\frac{u,v}{x,y}\right)$. Prove that $J\left(\frac{u,v}{x,y}\right) \times J\left(\frac{x,y}{u,v}\right) = 1$. 1+4

9. (a) A particle of mass m is falling under gravity. The resistive force from the medium is proportional to velocity of the particle. Set up the equation of motion of the particle and determine the velocity at a time t. Show its graphical variation. 1+3+1

(b) The position vector of a particle is: $\vec{r} = r\hat{e}_{r} + \hat{k}Z$. Determine the velocity and acceleration of the particle. 2½+2½

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3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Physics

Course : PHYS1021 (MINOR)

(Mathematical Physics-I)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols have their usual meaning.

Group-A

1. Answer *any five* questions from the following: 2×5=10

(a) What do you mean by limit of a function? Find the value of $\lim_{x\to 0} \frac{\tan x}{x}$.

(b) For a function $f(x) = \frac{(x+1)(x^2-25)}{(x-1)(5x^2+30+45)}$, list all values of x at which $f(x)$ is not defined.

(c) What is Wronskian of a differential equation? Show that if two solutions are linearly independent Wronskian vanishes.

(d) Find the projection of the vector $\vec{A} = 3\hat{\imath} + 3\hat{\jmath} - \hat{k}$ on the vector $\vec{B} = 2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$.

(e) If $\phi(x, y, z) = 3x^2y - y^2z + xz^2$, find $\vec{\nabla}\phi$ at the point (3, 1, –1).

(f) Determine the constant α so that the vector $\vec{A} = (2x + 3y)\hat{\imath} + (y - 2z)\hat{\jmath} + (3x + 2\alpha z)\hat{k}$ is solenoidal.

(g) Evaluate $\iint_S \vec{F}.\hat{n}\, dS$, where $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and S is the surface bounded by a unit cube.

(h) Solve the differential equation $\frac{dy}{dx} + 2y = 3e^x$.

Group-B

Answer *any two* questions from the following: 5×2=10

2. State the Stokes' theorem in words. Verify Stokes' theorem for $\vec{A} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 2+3

3. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time.

(a) Determine its acceleration at any time.

(b) Find the magnitudes of the velocity and acceleration at $t = 0$. 2+3

4. Obtain the scale factors for the cylindrical constants. Show that the cylindrical coordinates are orthogonal coordinate system. 3+2

5. State the existence and uniqueness theorem for initial value problems of ordinary differential equations. A tank initially contains 50 gallon of water. A salt solution containing 2 kg of salt per gallon water is poured into the tank at a rate of 3 gallon/minute. The mixture is stirred and is drained out the tank at the same rate. Find the initial-value problem that describes the amount Q of salt in the tank at any time. 2+3

Group-C

Answer *any two* questions from the following: 10×2=20

6. Define the irrotational vector. Find constants a, b, c so that a vector $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - 2z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$, where n is a constant. 2+2+6

7. What do you mean by linear differential equations? What is complementary function and particular integral? Solve the differential equation $\frac{d^2y}{dx^2} - 4y = x \sin hx$. 2+2+6

8. What do you mean by vector field? A vector field $\vec{F}$ is given by $\vec{F} = xy^2\hat{\imath} + 2\hat{\jmath} + x\hat{k}$ and L is a path parameterized by $x = ct, y = c/t, z = d$ for the range $1 \leq t \leq 2$. Evaluate the three integrals

   (a) $\int_L \vec{F}dt$

   (b) $\int_L \vec{F}dy$

   (c) $\int_L \vec{F}.d\vec{r}$ 1+3+3+3

9. Plot the function $y = x^2 + x - 1$ for $|x| < 5$. What is an exart differential equation? Show that the differential function $u(x,t) = f(x + ct) + \phi(x - ct)$ satisfies the differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$. 3+2+5

PHYS1051

3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Physics

Course : PHYS1051 (SEC)

(Renewable Energy and Energy Harvesting)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group-A

Answer *any five* questions: 2×5=10

1. (a) Explain how a solar cell generates power.

   (b) State the origin of geothermal energy.

   (c) Write down the principle of piezo electric effect.

   (d) What is the source of energy in stars?

   (e) Mention two names of water turbines used in hydroelectric power plant.

   (f) What is a linear generator?

   (g) How does a wind turbine affect radio signals?

   (h) What is a solar pond?

Group-B

Answer *any two* questions: 5×2=10

2. (a) Explain how a solar tracking system works.

   (b) Discuss the principle of a solar cooker. 2+3

3. (a) What is the importance of renewable energy?

   (b) Discuss the merits and demerits of renewable energy sources. 2+3

4. (a) What are the different types of geothermal resources?

   (b) Describe how heat can be extracted from geothermal sources. 2+3

5. (a) Discuss how the electromagnetic energy can be harvested.

   (b) What are the advantages of electromagnetic energy harvesting? 2+3

Group-C

Answer *any two* questions: 10×2=20

6. (a) What is the importance of solar energy? How can you store solar energy?

(b) Describe the working principle of a photovoltaic module.

(c) What is a solar green house? (2+2)+4+2

7. (a) What are the merits and demerits of ocean energy harvesting?

(b) How can the tide energy be stored?

(c) Explain the terms: 'Osmotic Power' and 'Ocean Bio-mass' 3+3+(2+2)

8. (a) Mention the defining characteristics of piezoelectric effect.

(b) Which type of materials are suitable for observing the phenomenon of piezoelectricity?

(c) Describe a model of piezoelectric generator with piezoelectric parameters. 3+3+4

9. (a) What is biochemical conversion?

(b) Explain how biogas can be generated.

(c) How can the wind energy be stored?

(d) On which factors does the power output of a wind turbine depend? 2+3+2+3

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